**UNIT V**

5. **INTEGRATION**

**5.1 Introduction**

In this unit, we will be looking at integrals. Integrals are the third and final major topic that will be covered in this section. As with derivatives this unit will be devoted almost exclusively to finding and computing integrals. Applications will be given in the following unit. There are really two types of integrals that we’ll be looking at in this unit: Indefinite Integrals and Definite Integrals. The first half of this unit is devoted to indefinite integrals and the last half is devoted to definite integrals. As we will see in the last half of the unit if we don’t know indefinite integrals we will not be able to do definite integrals.

**Unit Objectives**

At the end of this unit you will be able to:

* Understand the general concept of integral calculus.
* Identify different techniques of integration.
* Integrate numerous functions.
* Relate integral with derivative of function.
* Perform the calculation of area bounded by some simple curves.
* Derive the volume formula of sphere.
* Apply the knowledge of integral calculus in their life.
* Understand the fundamental theorem of calculus I and II.
* Compute the area bounded by simple curves.

**5.2 Antiderivative and Indefinite Integral**

The word "integral" may also refer to [antiderivative](http://en.wikipedia.org/wiki/Antiderivative) in a mild abuse of language. (The antiderivative of a function is that function whose [derivative](http://en.wikipedia.org/wiki/Derivative) is equal to the first function.) Though they are closely related through the [fundamental theorem of calculus](http://en.wikipedia.org/wiki/Fundamental_theorem_of_calculus)

(which is to be discussed later), -the two notions are conceptually distinct. When one wants to clarify this distinction, an antiderivative integral is referred to as an [**indefinite integral**](http://en.wikipedia.org/wiki/Antiderivative) (a function), while the integrals discussed in this article are termed **definite integrals.**

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| **Definition 5.1**  Let f be a continuous function on its domain.  A function *F* ( *x*) is called antiderivative of *f* (*x*) if and only if for all x in the domain of *f*  *F*(*x*) = *f* (*x*). |

**Example 1**.Consider f(x) = x, domain of f is the set of real numbers

Then F(x) = , where is an antiderivative of f.

i.e., F(x) = x.

Observe that the antiderivative of a given function is not unique; for instance

x2, x2 + 5, x2 – 4 and x2 + c, for c is arbitrary constant are all antiderivatives of

f(x) = 2x.

Since.

|  |
| --- |
| **Definition 5.2**  If *F* (*x*) is any anti-derivative of *f* (*x*), then the most general anti-derivative of *f* (*x*) is called an **indefinite integral** and denoted, ∫*f* (*x*)*dx* = *F* ( *x*) + *c*, *c* is any constant  In this definition, the ∫ is called the **integral symbol**, *f* (*x*) is called the **integrand**, *x* is called the **integration variable** and the “*c*” is called the **constant of integration**. |

Thus, we write 

**Example 2:** For x0, the function F(x)  is an antiderivative of f(x) = for an arbitrary constant number c. Thus 

**Example 3:**

For an arbitrary constant number c the function F(x) = Sin x +c is an antiderivative of the function f(x) = Cos x , for all x .

**Example 4:**

Since =

F(x) =  is an antiderivative of f(x) =xn for n- 1 and an arbitrary constant c moreover, F(x) = ln is an antiderivative of f(x) =

|  |
| --- |
| **Theorem 5.1**  Let f be continuous on [a, b]and F and G are antiderivative f on [a, b],then there exist  c such that,  F(x) = G(x) + c |

**Proof:**

We assume that F(x) and G(x) are not identically equal to each other.

Thus, F(x) = f(x) = G(x) 

F(a) = f(a) = G(a) ,

F(b) = f(b) = G(b)

F and G are continuous on [a, b] and differentiable in (a, b)



Thus, by corollary 1of mean value theorem, F – G is a constant function



By the above theorem, one can conclude that every continuous function has an infinite number of antiderivative and any two of them only differing by a constant.

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| **Theorem 5.2**  If F and G are antiderivative of the continuous functions f and g respectively, and c is any constant then   1. cF is an antiderivative of cf 2. F+G is antiderivative of f + g 3. F – G is antiderivative of f – g |

**Proof** of (iii)

Since (F –G)= F– G= f – g it follows that F – G is antiderivative of ( f – g ).

Similarly you can verify the remaining.

**Example:** Find the antiderivative of f(x) = 3x2 – 8x3 – 6x + 2

**Solution:**



  is antiderivative of 3x2 – 8x3- 6x+2.

**Example:** Show that the polynomial; Q(x) =  is an antiderivative of the polynomial P(x) where P(x) = 

Since, is an antiderivative of the function ajxj for j = 0,1,2,3, . . .,n. It follows that Q(x) is an antiderivative of P(x).

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| **Definition 5.3**  The set of all antiderivative of a continuous function f is called the Indefinite Integral of f and is denoted symbolically by. The function f is called the integrand and x is called the variable of integration. |

Thus , if f(x) is an antiderivative of the continuous function f and c is any constant ,then the relation F(x) +c holds and previous theorem can now expressed as follows;

f(x) dx = k.

ii).

iii) 

The process of finding the set of antiderivative of the continuous function f is called Integration of the function f with respect to the variable of integration x.

**5 .3 Integration of Simple Trigonometric and Exponential Functions**

* + - 1. **Integration Formulas**

A number of formulas below follow immediately from the standard differentiation formulas of earlier unit.

 2)

 4) 

 6) 

7) 

**Exercises 5.1**

Integrate the followings

1)  2) 

3) 4) 

5)  6) 

7)  8) 

9)  10) 

11) 12) 

13)  14) 

15)  16) 

17) 18)

19) 20) 

**5.4 Techniques of Integration**

There are some important methods to evaluate indefinite integrals which we call techniques of integration.

1. **Integration by substitution**

Let’s see integration by substitution by examples.

**Example 1:** Suppose we want to find the antiderivative of 

One can find the antiderivative of (x +2)7 by expanding in to a polynomial of degree 7 and then integrate term wise using the formula 

However, this method for large degree is cumbersome and tedious; thus using integration by substitution method is preferable.

Therefore, to find the result substitute u for (x+2)

Now if u = (x+2)

* du = dx,

Then the variable of integration **x** will be replaced by **u.**

Hence, the given indefinite integral has the form;



Now we can use the above () to get the antiderivative of the given function



And substituting back the value for u, we have the result



**Example 2**: Find 

**Solution**: Let u = 5x

* du = 5dx
* 

Then, 

= 

= 

|  |
| --- |
| **Theorem 5.3**  If for all x a ,b, the function g(x) is continuous and f(u) is continuous at g(x) ,then |

**Proof:**

If F(u) is an antiderivative of f(u), then we have



i.e., converting the chain rule of differentiation in to method of integration by substitution

F (g(x)) is an antiderivative of f (g(x)).g’(x)

Thus we have;



Now since u = g(x) , then 

**Example 1:** Integrate 

**Solution:**

Let g(x) = x5 + 6 = u

 g(x) = 5x4dx = du

Now, 

= , but u = x5 +6

= 

Therefore,  = 

**Example 2:**

Integrate 

**Solution:** Let u = 4x +

du = 2(2+x)dx

du/2 = (x+2) dx

 =  =  =

=  = 

, = 

**Example 3**: 

**Solution:** 



= 

**Example 4**: Integrate

**Solution:** Let u =1 +

du =

Therefore,  = =

= 

**Exercises 5.2**

Integrate the followings

1.  2.  3. 

4.  5.  6. 

7.  8 .  9. 

10.  11.  12. 

13.  14. dx 15. 

16.  17. 

1. **Integration by parts**

As we converted the chain rule of differentiation in to the method of integration by substitution, in this section we will use the product theorem for differentiation in terms of integral.

The resulting integration theorem will allow us to evaluate integrals of the form such as;



|  |
| --- |
| **Theorem 5.4 ( Integration by parts)**  Let f and g be functions with continuous derivative on their domains; |

**Proof**:

Using the product rule for differentiation,

i.e.,

Since f, g, fand gare continuous the indefinite integrals

, which is reduced to;



 = f(x) g(x) - 

**Example 1: Integrate**

**Solution:** Let f(x) =x and g(x) =cosxdx

f(x) =1.dx, g(x) = sinx and using the above theorem consequently,

 = xsinx - 

= xsinx – (-cosx) +c

= xsinx +cosx +c.

**Example 2:** Find 

**Solution:**

Let f(x) = x2 

Thus 

Again let 







 = 

**Example 3:** Evaluate

**Solution:**

Let f(x) =lnx  = dx 

 = xlnx -

= 

=

**Example 4**: Find 

**Solution:** Let f(x) = ln x g(x) = xdx







**Example 5:** Find 

**Solution**: Let 



 =

Again let  and 

 and 

As a result,  =

= 

 +  = 

**Exercises 5.3**

Find the indefinite integrals of the following by using integration by parts method

1) 2) 3) 

4)  5)  6) 

7)  8)  9)  10) 11) 

1. **Integration by partial fraction**

Consider a rational function R(x) which is a quotient of two polynomials. That is



If it is not expressed in that way, we must change the given expression in to the desired property by using long division

**Example**1: 

Therefore, the rational function R(x) is the sum of the polynomials (x2+1) and the rational function. Where the degree of the numerator is less than the degree of the denominator.

In this case, we shall give a systematic method, known as a partial fraction of integrating rational functions.

For the method we need the support of the following theorem which we shall accept without proof.

|  |
| --- |
| **Theorem 5.5**  If two polynomials in the same variable x are identical, then the corresponding powers of x have the same coefficients. |

That is, if 4x3 +3x + 9 = Ax4+Bx3+ cx2+Dx +E for all x, then

A= 0, B = 4 , C = 0 D= 3 E = 9

We engage this method only for the following cases:

1. Q(x) factors into distinct linear factors
2. Q(x) factors in to repeated linear factors.

Integration of rational function of  by the method of partial fraction consists essentially of two steps, namely:

1. Factorizing Q(x) and
2. Expressing the rational function as a sum of simple rational functions whose denominators are linear factors.

**Example 1:** Integrate 

**Solution :**

The integrand is 

Thus, factoring (x2 - 9) and we obtain 

Therefore, we express as a sum of simple rational functions to get

constants to be determined

This gives, 













=

= 

= 



**Example 2**: Evaluate;

**Solution:**

By long division method we obtain;

Now first treat the rational part thus;

Let 









= 

=

= =

  = 

**Example 3:** Evaluate****

**Solution:**

Let 

**** 

=  =

** =** 

**Example 4:** Evaluate ****

**Solution**:

For R(x)  the degree of P(x) >the degree of Q(x), so we use the long division method to express the given integrand





= 





=  =

,



Also A+C = 4 

 = 



= 

Therefore, **** = 

= 

= 

= 

**Exercises 5.5**

Integrate the following

1. 2. 3. 

4.  5. 6.

7. 8. 9. 

10.  11.  12. 

13. 14.  15. 

16.  17. 18. 

19.  20.

1. **Integration of trigonometric functions**

There are integrals involving trigonometric functions which cannot be evaluated easily using the methods already developed.

However, such integral can be evaluated by transforming the integrand by means of trigonometric identities such as;





=

Similarly, sin2x 

**Example1:** Evaluate 

**Solution:** Using the above identity

 =  

=  + 

 = 

= 

 = .

**Example 2:** Find 

**Solution:** By using trigonometric identity, we have

 = 

=  = 

= 

 = 

**Example 3:** Evaluate 

**Solution:**

 =  = 

Now let u= sinx, then du = cosxdx (by making substitution)

 =

= 

= 

Therefore,  = 

**Example 4:** Evaluate 

**Solution:**

 =  . Now we let u =cosx , then du = -sinxdx

 = 



 = 

**Example 5:** Evaluate 

**Solution:** We factor out cos x and write the rest





Now we let u = sinx so that du =cosxdx. Thus, we obtain

 = 

= 

 = 

**Exercises 5.5**

Integrate the following

1)   3) 

4)  5)  6) 

7)  8 )  9) 

**Miscellaneous substitutions**

Sometimes we need a combination of the above discussed techniques to evaluate a given functions.

To show this we have some examples

**Example 1:** Find 

**Solution**

Using substitution method; Let 



Thus,  = 

Let f(x) = t , f(x) = dt and g(x) = etdt then g(x) = et .

Thus 



**Example 2**: Find 

**Solution:**

Now let us apply integration by parts first;

 = , then let 



 = ;

Next we use substitution method to evaluate , If u = x2  then du =2xdx  

**Example 3:** Find

**Solution:**

To find the solution, first we use substitution method

Now we let 

 = ; Now using partial fraction method on 

 =   



  =  = = 

=

  = 

For the case 

**Example4:**

**Solution**:

Now apply partial fraction

i.e,

 2A = 1

= +c = 

=

**Example 5:** Evaluate 

**Solution:** Now, let x+2 = u2  

 = 

Next apply partial fraction method Let 



  = 



= 

**Exercises 5.6**

Evaluate the following integrals

1)  2)  3) 

4)  5)  6) 

7)  8)  9) 

10) 11)  12) 

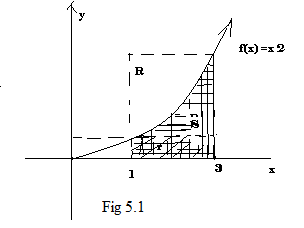
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**5.5 The Definite Integral and fundamental theorem of calculus**

Our goal in this section is to show how area under a curve, such as the area A of a rectangle, of triangle so on can be expressed as a limit of a sum of terms which is called a definite integral.

We will then introduce a result called the fundamental theorem of calculus that allows to compute definite integrals and thus find area and other quantities by using indefinite integration (antiderivative) methods of the (previous section).

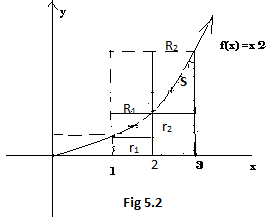
Now, consider the region S in fig5.1 which is bounded above by f(x) = x2, below by the x-axis from x = 1 to x=3.

****

Let R and r be rectangles with the same base on the x-axis of closed interval [1, 3] enclosing S and enclosed by R respectively, then

A( r) A(S) A(R), where A(r ),A(S) and A(R ) are the area r, S, and R respectively .If we substitute the given interval [1, 3] in to two equal sub-intervals [1, 2] and [2, 3]and if we denote by R1 and R2 the rectangles with bases [1,2] and [2,3] enclosing S and by r1 and r2 the rectangles on bases [1,2] and [2,3] enclosed by the region S as shown figure

5.2 below we have A (r1) + A (r2) A(S) A (R1) +A (R2).



If we subdivide the interval [1, 3] in to four equal sub divisions [1, 3/2], [3/2, 2], [2,5/2] and [5/2, 3]and if we denote by R1, R2 , R3, and R4 the rectangles enclosing S and by r1, r2 , r3 , and r4 the rectangles enclosed by S we have

A (r1) +A (r2) +A (r3) +A (r4) A(S)A(R!) +A (R2) +A (R3) +A (R4)

****

If we continue sub-dividing the interval [1, 3] in to n equal subintervals

1 x1 x2 x 3 . . . x i -1 xi xi+1 . . . xn-1  xn=3

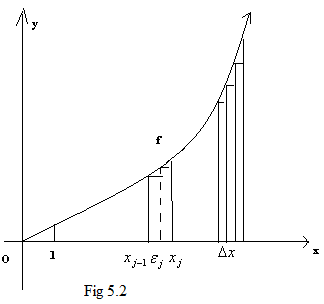
and denoting the enclosing and the enclosed rectangles on each subintervals by Ri and ri respectively we find 

Therefore, we can observe that as the number of equal sub intervals increase the length of each subinterval decreases or tend to zero.

This small sub-interval is denoted and = x and hence the difference

and eventually approaches zero.

That is, 



Since,  and

, we can approximate A (Rj) and A(rj) by the expression where  is any point in the subinterval and thus write



 Using the above illustration of the notion of area under a curve, we associate the idea with the concept of definite integral as follow;

1. **The Definite Integral**

Let f(x) be a function that is continuous on the interval  Subdivide the interval

in to n equal parts using the points a= x0 , x1, x2, . . . x j-1, xj, xj+1, . . .,xn =b and the subdivided interval [a, b] using the points;

[a, x1] ,[x1, x2] [x2 ,x3] . . . [xj, xj+1] . . . [xn-2,  xn-1], [xn-1 , xn] each of width  and choose a number xj from the jth subinterval for j = 1, 2, 3, . . . n.

For the sum, [ f(x1) + f(x2) + . . . f(xn)] called an integral sum or Riemann sum of f in the interval [a, b].

Then the definite integral of f on the interval is the limit of the Riemann sum as That is:



= 

The function f(x) is called the integrand, and the numbers ***a***and ***b*** are called the lower and the upper limits of integration respectively.

The process of finding the definite integral is called definite integration.

1. **Area as a Definite Integral**

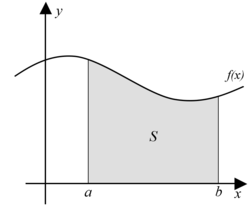
[](http://en.wikipedia.org/wiki/Image:Integral_as_region_under_curve.png)

Fig 5.3

Suppose f is continuous and f(x) 

If the limit of the integral sum  and equal to the real number S, then the number S is called the definite integral of f over the interval [a, b] and is denoted by; S =  is read as “the integral of f with respect to x from a to b.’’ and we say that f is integrable.

The symbol  used for definite integral is essentially is the same as the symbol for the indefinite integral, even though the definite integral is a specific number

While the indefinite integral is a family of functions, the antiderivative of f.

It is not quite evident from the definition of the definite integral whether the limit of integral sum  exists for all functions.

We shall now state a theorem without proof which assures us when a function is integrable.

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| **Theorem 5.6 (Existence of definite integral)**  If the function f is continuous on [a, b] ,then  That is, f is integrable over [a, b]. |

**Example 1:** Let f(x) = cos(x) then find the integral f over 

**Solution:**

Since cos(x) is continuous in, then the integral of exists.

= sinx = 

**Example 2:** Let p(x) be any polynomial function, then find the integral of p (x) over.

**Solution:** Since any polynomial p(x) is continuous in the finite interval [a, b], the integral 

**Example 3:** Let, then find definite integral of f over 

**Solution:** Since the function f(x) = is not defined at x = 0;and 0 [-1,1] the integral does not exists.

**Example 4:** Use the definition of definite integral to show thatwhere k is any constant.

**Solution:** Subdivide the given interval [a, b] in to n equal subintervals each of length

x =

That is [a, x1] ,[x1, x2] ,[x2, x3] . . .[xj-1 , xj] . . . [xn , b]

Since f(x) = k we have  consequently we have





**Exercise 5.7**

1. Let S be the region bounded by f(x) = x3, the line x =2,the x axis and x = 4 and ;

Estimate A(S) by subdivided the interval [2, 4] in to

a) 2 equal subintervals b) 4 equal subintervals

2. Let S be the region bounded by f(x) = sinx, x = 0, the x axis and x = 

Estimate A(S) by subdividing the interval [0, ]

a) 2 equal subintervals b) 4 equal subintervals.

3. Determine whether the following functions are integrable or not in the given interval

a)cosx , [-,] b) sin3x , [-/2 , /2 ]

c) tanx , [0 , /2] d) tanx [0 , /4]

e) sinx +cosx , [0, /2] f) 1/x , [1/2 , 100]

g) 3x2 +7x -2, [ -1 ,1] h) 

The following properties of definite integrals are immediate consequences of its definition and can be used to simplify the computation of definite integrals.

|  |
| --- |
| **Theorem 5.7**  If f and g are integrable over [a, b] and k is any constant then kf, f +g, f – g are integrable on . And  1)  2)  3) )  4)  5) If f is integrable on and c[a, b] then  ; Subdivision rule  6) Let f is integrable over [ a , b]  If m f(x)  M for all x ,then m(b-a) |

**Proof of 1**:

Since  it follows that kf is integrable over [a,b] and 

**Proof of 2:**

Since f and g are integrable over [a, b]  exists where x1 and x2 are the length of the subintervals of [a, b]corresponding to the functions f and g and are any two elements in the jth subinterval corresponding to the functions f and g. Let 

 and hence fg is integrable in [a ,b]

Hence 

**Proof of 5:**

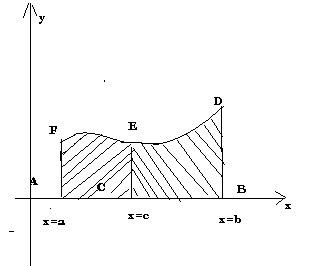
We assume that we can always subdivide the interval [a, b] in to n equal subintervals with c as one of the end points of a subinterval [a, b] = [a, c] and [c, b] respectively then 

Since f is continuous on [a, b] and hence on [a, c] and [c, b].

Therefore, each of the integral sums in tends respectively to 

Since  By the limit theorem we have. Therefore, property (5) has geometrical interpretation when f(x) is positive as seen in the figure below.

Therefore, clearly the area of the region ABDF is the sum of the region of ACEF and CBDF.

Fig5.4

for

**Proof of property 6:**

Since  we have by property (4)

 and by property (3)







Similarly, 

 Combining

|  |
| --- |
| **Theorem 5.8 (Mean Value Theorem for Integrals)**  If f is continuous on the interval [a, b] , then there is a number x0 between a and b such that . |

**Proof:** Since f is continuous on [a, b] there are numbers x1and x2  in [a, b] such that

f(x1) = m , f(x2) = M and 

By property (5) of theorem 5.7 we have:

Thus, there is a number such that:

. And by Intermediate Value Theorem, there is a number

]  and hence; 

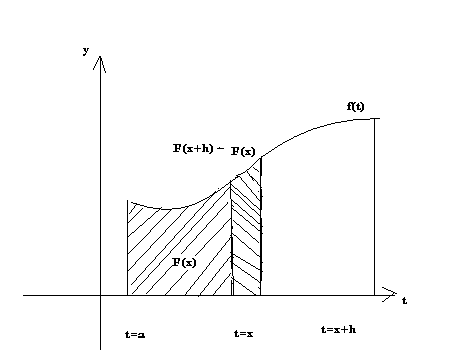
**5.6 The Fundamental Theorem of Calculus**

The purpose of this section is to develop a general method for evaluating  that does not require computing various sums.

Thus, the method will allow us to evaluate many (but not all) of the integrals that arise in applications.

Let us consider the definite integral  for a continuous function f(t) defined on [a, b], where the lower limit of integration is fixed, that is constant, and the upper limit varies within the interval [a, b].

Since for each  the definite integral  is a fixed number, it is clear that F(x) is a function.

For positive function f(t) the function F(x) represents the area of the region bounded by the graph of f(t) , t = a , the x-axis and t = x Fig5.5

|  |
| --- |
| **Theorem 5.9 (Fundamental Theorem of Calculus I (FTCI))**  If the function f(t) is continuous in [a, b] and for then F(x) is differentiable on [a, b] and |

**Proof:**

Suppose x and x+h are in (a, b). We then have:

**



.

Finally, assume that  and we get

 ………………(1)

Let’s now assume that and since we are still assuming that x and x+h are in (a, b) we know that f(x) is continuous on  and so by the extreme value theorem we know that there are numbers c and d in  so that f(c) = m is the absolute minimum of f(x) in  and that f(d) = M is the absolute maximum of f(x) in .

So, by the integral properties we then know that we have,

 or 

Now divide both sides of this by h to get



and then use (1) to get, ……….(2)

Next, if  we can go through the same argument above except we will be working on [x+h, x] to arrive at exactly the same inequality above. In other words, (2) is true provided .

Now, if we take we also have  and because both c and d are between x and x+h. this means that we have the following two limits.

 and 

The Squeeze Theorem then tell us that,

…………………………(3)

But, the left side of this is exactly the definition of the derivatives of F(x) and so we get that .

 F(x) is differentiable on the interval (a, b).

Fundamental theorem of Calculus (I) enables us to find a function F(x) whose derivative is the given function f(x) .In other words, the Theorem asserts that every continuous function on [a, b] is a derivative of some function.

|  |
| --- |
| **Theorem 5.10: (Fundamental Theorem of Calculus(II)(FTCII))**  Let f(t) be continuous on [a, b]. If G(t) is an antiderivative of f(t) on [a, b], then |

**Proof**: First let  and then we know by FTC(I) that  and so F(x) is an anti-derivative of f(x) on [a, b]. Further suppose that G(x) is any anti-derivative of f(x) on [a, b] that we want to choose. So, this means that we must have,



Then, by theorem 5.1 F and G differ by a constant on [a, b]. In other word for  we have,



Now because F(x) and G(x) are continuous on [a, b], if we take the limit of this as  and  we can see that this also holds if x= a and x= b.

So, for we know that . Let’s use this and the definition of F(x) to do the following.



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= 

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Note that in the last step we used the fact that the variable used in the integral does not matter and so we could change the t’s to x’s.

This Theorem answers in full the problem of evaluating definite integrals.

is to find another function whose derivative is f(x).

After that one only does a simple calculation

Notation  to denote the difference G(b) –G(a).

From the fundamental theorem, we can now conclude that if f(x) > 0, then the area of the region bounded by f(x), x-axis, x = a and x = b is



**Example 1:** Evaluate 

**Solution:** We know that F(x) = x2 is antiderivative of f(x) = 2x



**Example 2**: Evaluate 

**Solution**: we know that F(x) =1/x2  is an antiderivative of ,



**Example 3**: Evaluate 

**Solution**: we know that F(x) = -cosx is an antiderivative of the function sinx

  =  = 

**Warning**

We must be sure that the integral is continuous between the limits of integration in order to apply the fundamental Theorem of Calculus ( II )

For example, we cannot use the FTC to evaluate , since -2/x3 is not continuous at x = 0 as .

**Example 4:** Evaluate 

**Solution:** Since is an antiderivative of the function anxn for n,and by the properties of integral we have;

 = 

**Example 5:** Find the area of the region bounded by f(x) = x4, x =2, x = 3 and the x-axis

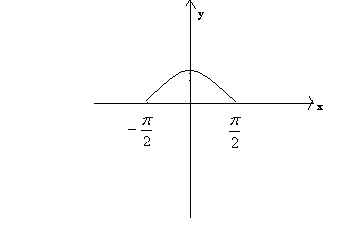
**Solution:** Since f(x) > o for , then the area of the region is

 = 

**Example 6**: Find the area of the region bounded by ; f(x) = cosx,

x = 

**Solution**: Since cosx0 for  the area of the region is

Fig5.6



**Exercises 5.8**

I-In the following exercises, use the Fundamental Theorem of Calculus (I)to differentiate the given function of x.

1) f(x) =  2) g(x) = 

3) h(x) =  4) k(x) = 

II – In the following, evaluate the given integrals.

1)  2)  3)

4)  5)  6)

7)  8)  9 ) 

10)  11)  12) 

**5.7 Change of Variables**

If for each tis continuous and the function f is continuous at g(t), then by the method of substitution we have:

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If we integrate the right hand side from a to b ; Since u = g(t), the limit of integration of the left hand side of above will be u = g(a) and u = g(b) and hence .

**Example 1** : Evaluate

**Solution:** Let g(x) = cosx and f(u) =√u

**Example 2:** Evaluate the integral: 

**Solution**  Let u = lnx; du =  

**Exercises 5.9**

Evaluate the following definite integrals.

